Microwave Measurement of Conductivity and Permittivity of Semiconductor Spheres by Cavity Perturbation Technique

ABHAI MANSINGH AND ANAND PARKASH

Abstract—Simple analytical relations for evaluating the components of complex relative permittivity of semiconductors using a cavity perturbation technique for spherical samples are presented. The relations although derived under a simplifying approximation yield results of almost the same accuracy as obtained by computer solutions of a transcendental equation for samples with resistivity up to about 1 Ω·cm.

I. Introduction

RESONANCE METHODS based on cavity-perturbation technique have been widely used for the measurement of permittivity (ϵ) and conductivity (σ) of materials at microwave frequencies [1]–[6]. However for samples of high conductivity, measurement of ϵ is difficult [2], but σ can be accurately measured using the technique based on the "Eddy-Current Loss" method [5], [6]. The procedure employed for evaluating ϵ and σ from the cavity measurements for semiconductors has been described by Champlin *et al.* [2] for spherical samples. The dielectric parameters are obtained by first calculating the "effective" parameters and then solving a cumbersome, complex, and transcendental equation. The solution of such an equation requires essential use of a fast computer and the application of the technique of interpolation. This makes the method unsuitable for quick evaluation of dielectric parameters of samples. A need therefore exists to simplify the method of evaluating ϵ and σ from the measured parameters.

In the present work, the method of solving the transcendental equation has been simplified making it possible to obtain accurate results using just a desk calculator for a substantial part of the range of ϵ and σ for which the Champlin's method can be employed. The simplified method has been used to calculate the relative permittivity ϵ_r and the relative conductive loss σ_r from the data of effective values reported by Champlin *et al.* The calculated values show excellent agreement with those obtained by solving the complex transcendental equation. In addition these equations have been used to evaluate the dielectric parameters of Si (resistivity, $\rho \cong 100$ Ω·cm) and Ge ($\rho \cong 10$ Ω·cm) from measurements made using a cylindrical TM_{010} mode cavity resonating at 3.6986 GHz. The results for ϵ_r and σ_r show good agreement, respectively, with the literature values and the dc conductivity of the samples.

Manuscript received April 22, 1980; revised August 28, 1980.

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II. Theory

A. Relations for Evaluating the Relative Permittivity and Conductivity of Spherical Semiconductors

When a small spherical specimen is inserted in a resonating cavity, in a position where the magnetic field is zero, the resulting relative change in the complex resonance frequency of the cavity can be expressed as [2]

$$\delta\Omega/\omega_0 = 1.5\ V_1(1-\epsilon^*_{r(\text{eff})})/C_c V_0(2+\epsilon^*_{r(\text{eff})}) \tag{1}$$

where C_c is the cavity constant, V_1 and V_0 are, respectively, the volume of the specimen and that of the cavity. $\epsilon^*_{r(\text{eff})}$ is the "effective" complex permittivity of the specimen and is related to the actual complex relative permittivity $\epsilon^*_r = \epsilon_r - j\sigma_r$ through the relation

$$\epsilon^*_{r(\text{eff})} = \epsilon^*_r F^*(\beta^* R) = \epsilon_{r(\text{eff})} - j\sigma_{r(\text{eff})}. \tag{2}$$

The function $F^*(\beta^* R)$ is given by

$$F^*(\beta^*R) = 2\mu(\beta^*R)/\{2\mu(\beta^*R) + (\beta^*R)\mu'(\beta^*R)\} \tag{3}$$

with

$$\mu(\beta^*R) = \{\beta^*R\cosh(\beta^*R) - \sinh(\beta^*R)\}/(\beta^*R)^3 \tag{4}$$

and

$$\beta^*R = j\omega_0(\mu_0\epsilon_0)^{1/2}(\epsilon^*_r R^2)^{1/2} \tag{5}$$

where R is the radius of the specimen and ϵ_0 and μ_0 are, respectively, the permittivity and permeability of free space.

We define

$$\delta\Omega/\omega_0 = (\delta\omega/\omega_0) + (j/2)\{(1/Q_1) - (1/Q_0)\} \tag{6}$$

where Q_1 and Q_0 are, respectively, the quality factors of the cavity with and without the sample, $\omega_0 = 2\pi f_0$ is the angular resonance frequency of the empty cavity, and $\delta\omega$ is the change in ω_0. The separation of (1) taken along with (2), into real and imaginary parts gives

$$\epsilon_{r(\text{eff})} + 2 = 3X/(X^2+Y^2) \tag{7}$$

$$\sigma_{r(\text{eff})} = 3Y/(X^2+Y^2) \tag{8}$$

where

$X\equiv 1+(1/K)(\delta\omega/\omega_0) \qquad Y\equiv(1/K)\delta(1/2Q)$

$K\equiv 3V_1/(2C_cV_0)$ and $\delta(1/2Q)\equiv(1/2)\{(1/Q_1)-(1/Q_0)\}$.

The components of ϵ_r^* have so far been obtained by calculating $\epsilon_{r(\text{eff})}^*$ from the known values of $(\delta\omega/\omega_0)$, $\delta(1/2Q)$, V_1, V_0, and C_c and then by solving the complex transcendental equation (2). However for most cases the calculations for the components can be simplified since (5) can be expressed as

$$\beta^*R=j\beta_0\epsilon_r^{*1/2}\equiv jZ \tag{9}$$

with

$$\beta_0=\omega_0(\mu_0\epsilon_0)^{1/2}R=2\pi f_0R/C$$

where C is the speed of light. Moreover the spherical Bessel function $j_1(Z)$ of the first kind can be expressed as [7]

$$j_1(Z)=(\sin Z-Z\cos Z)/Z^2. \tag{10}$$

These two expressions when combined with the following set of equation [8]:

$$\cosh(jZ)=\cos Z$$
$$\sinh(jZ)=j\sin(Z) \tag{11}$$

permit to write (4) as

$$\mu(\beta^*R)=j_1(Z)/Z \tag{12}$$

from which it follows that

$$(\beta^*R)\mu'(\beta^*R)=-j_2(Z). \tag{13}$$

Substitutions from (12) and (13) in (2) taken along with (3) lead to

$$\epsilon_r^*/\epsilon_{r(\text{eff})}^*=1-(Z/2)\cdot\{j_2(Z)/j_1(Z)\}. \tag{14}$$

Relation (14) is exact and holds good for all permissible values of $Z=\beta_0\epsilon_r^{*1/2}$ for which the basic assumptions of the cavity-perturbation technique are valid. As such it is a complex transcendental equation and is therefore not suitable for quick evaluation of ϵ_r^*. Replacing the spherical Bessel functions by their series expansion given by [9]

$$j_n(Z)=j_n(\beta_0\epsilon_r^{*1/2})=\epsilon_r^{*1/2}\sum_{k=0}^{\infty}(-1)^k(\epsilon_r^*-1)^k\cdot(\beta_0/2)^kj_{n+k}(\beta_0)/k! \tag{15}$$

and approximating the sums up to the terms containing Bessel functions of second order, (14) becomes

$$\epsilon_r^*/\epsilon_{r(\text{eff})}^*=1-\beta_0\epsilon_r^*j_2(\beta_0)/2\{j_1(\beta_0)-0.5(\epsilon_r^*-1)\beta_0j_2(\beta_0)\}. \tag{16}$$

For most cases (16) is not likely to cause significant error as β_0 is generally less than one and $j_3, j_4,\cdots$, are much less than j_2. Separation of real and imaginary parts in (16) leads to

$$\sigma_{r(\text{eff})}=\sigma_r\left[2-A^2/\{2B^2(\epsilon_r^2+\sigma_r^2)+A(A-2B\epsilon_r)\}\right]^{-1} \tag{17}$$

$$\epsilon_{r(\text{eff})}=(\sigma_{r(\text{eff})}-\sigma_r)(A-2B\epsilon_r)/2B\sigma_r \tag{18}$$

where

$$A\equiv 2j_1(\beta_0)+\beta_0j_2(\beta_0) \tag{19a}$$

and

$$B\equiv\beta_0j_2(\beta_0). \tag{19b}$$

For a given set up and the radius of the sample, A and B are constant as is evident from (9) and (19). Relations (17) and (18) can also be expressed as

$$\sigma_r=\sigma_{r(\text{eff})}\left[1-2B\epsilon_{r(\text{eff})}/\{A-2B(\epsilon_r-\epsilon_{r(\text{eff})})\}\right] \tag{20}$$

and

$$B\epsilon_r^2-(A+2B\epsilon_{r(\text{eff})})\epsilon_r+\left(A\epsilon_{r(\text{eff})}-B\sigma_r^2+2B\sigma_r\sigma_{r(\text{eff})}\right)=0. \tag{21}$$

Since σ_r must be positive and $\sigma_{r(\text{eff})}$ cannot be negative (see (8)), it follows from (20) that

$$A\geqslant 2B\epsilon_r. \tag{22}$$

In principle (20) and (21) can yield values of ϵ_r and σ_r but (21) may yield four values of ϵ_r out of which the correct value will be the one which satisfies the inequality (22). For most samples it is however found that A is much greater than $2B(\epsilon_r-\epsilon_{r(\text{eff})})$ (see Table I). For such cases (20) can further be simplified to

$$\sigma_r=\sigma_{r(\text{eff})}\left[1-\{2B\epsilon_{r(\text{eff})}/A\}\right]. \tag{23}$$

σ_r calculated from (23) when substituted in (21) will lead to two values of ϵ_r out of which only the correct one will satisfy (22). The accuracy of the results can be increased by using the method of successive approximation in (20) and (21). However as shown later, results with error less than 2 percent are obtained directly from (23), (21), and the condition (22) without using any successive approximation.

III. Measurements

The experimental setup used for measuring the effective dielectric parameters of n-type silicon and n-type germanium with resistivity 100 Ω·cm and 10 Ω·cm, respectively, was the same as described earlier [10]. The measurement technique employed for measuring the changes in the resonance frequency δf and the quality factors of the cavity was the same as reported elsewhere [3]. The resonance frequency and the quality factor of the empty cavity were, respectively, 3.6986 GHz and 1849. The other measured parameters are given in Table II.

IV. Results and Discussion

Relations (17) and (18) have been derived assuming that in (15) terms containing the spherical Bessel functions of order more than 2 are negligible compared to those containing the Bessel function of lesser order. In order to see to what extent (17), (18), and hence (20), (21), and (23) yield accurate results, the relationship between the effective and the actual values of the dielectric parameters at 9.6 GHz, evaluated from (17), (18), and the exact relation (2) have been plotted in Fig. 1 for positive values of $\epsilon_{r(\text{eff})}$. Curves obtained from (17) and (18) have been shown by

TABLE I

ACTUAL DIELECTRIC PARAMETERS (ϵ_r AND σ_r) EVALUATED FROM THE EFFECTIVE VALUES USING THE EXACT TRANSCENDENTAL RELATION (2) AND THE SIMPLER SET OF RELATIONS (21) AND (23)

Sample	$a \times 10^2$ (cm)	$\beta \times 10^1$ (rad)	$A \times 10^1$	$B \times 10^4$	$\epsilon_{r(eff)}$	$\sigma_{r(eff)}$	From (2) ϵ_r	From (2) σ_r	From (21) and (23) ϵ_r	From (21) and (23) σ_r	
I n-type Si	9.45	1.8767	1.2505	4.0442	12.45	0.832	11.64	0.765	11.9	0.765	Work of Champlin et al. [2]
II p-type Si	10.26	2.0375	1.2583	5.6236	12.26	1.500	11.68	1.400	11.61	1.340	
III n-type Ge	11.52	2.2838	1.5224	7.8517	16.72	6.290	15.97	5.220	15.42	5.200	
IV n-type Ge	9.82	1.9502	1.2998	4.7584	13.97	22.59	15.6±1	20.53	15.10	20.30	
V n-type Si	10.33	2.0514	1.2673	5.5697	−1.34	62.99	15±5	60.91	17.70	63.70	
I n-type Si	15.60	1.2082	0.8182	8.8319	12.98	6.67	—	—	11.63	4.800	Present Work
II n-type Ge	15.88	1.2302	0.8201	1.2394	12.58	51.38	—	—	16.35±2.5	49.40	

TABLE II

MEASURED PARAMETERS AT ROOM TEMPERATURE (34°C) WITH A CAVITY OF $f_0 = 3.6986$ GHz AND $Q_0 = 1849$

Sample	ρ(dc) ohm m	$\sigma_r = \frac{1}{\rho_{dc}\,\omega\,\epsilon_0}$ at 3 6986 GHz	a mm	δf MHz	Q_1
I n type Si	1 0	4 87	1 560	4 50	1273
II n type Ge	0 1	48 7	1 588	5 32	1378

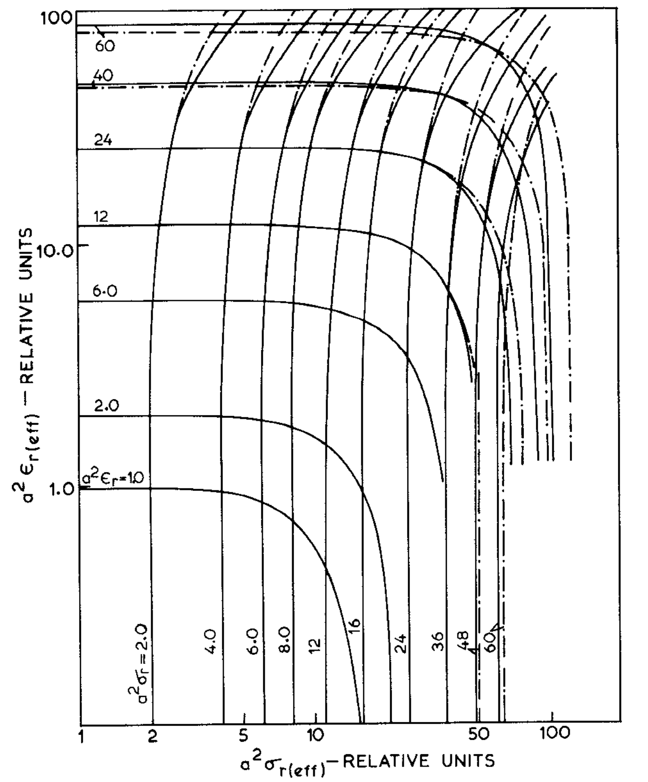

Fig. 1. Relationship between $\{\epsilon_{r(eff)}, \sigma_{r(eff)}\}$ and $\{\epsilon_r, \sigma_r\}$ at 9.6 GHz for a sphere with radius $= a$ mm, $\mu_r = 1$, and $\epsilon_{r(eff)} > 0$.

full lines while those obtained from (2), by broken lines. It is seen that for a sphere of a mm, the set of simultaneous equations (17) and (18), yield results almost as accurate as the exact relation if the modulus $|a^2\epsilon_r^*|$ is less than or equal to about 35. It implies that for a semiconductor of relative permittivity 15, (17), and (18), and hence (20) and (21) yield correct results for the entire range of resistivity from ∞ to about 6 $\Omega \cdot$cm for a specimen of radius 1 mm; and reducing the radius of the specimen would decrease the lower limit of the resistivity up to which the equations are valid. At other frequencies, the upper limit of $|a^2\epsilon_r^*|$ up to which (17) and (18) would be valid, is obtained by replacing the radius a by the "corrected" radius, $a' = \{f\ (\text{GHz})/9.6\}(a)$, defined by Champlin *et al.*, in the equation $|a^2\epsilon_r^*| = 35$. Thus at 3.2 GHz, the corrected radius would be one third of a while the upper limit of $|a^2\epsilon_r^*|$ would be 315 for which (17) and (18) are valid. It means for frequency equal to 3.2 GHz, $a = 1$ mm and $\epsilon_r = 15$, the set of simultaneous equations is as accurate as the exact relation up to resistivity about 2 $\Omega \cdot$cm.

The analytical relations (17) and (18) which simplify the process of evaluation of the dielectric parameters have been studied for the negative range of $\epsilon_{r(eff)}$ also. It has been found that these are valid for the range $0 \leqslant |a^2\epsilon_r^*| \leqslant 80$, implying thereby that at 9.6 GHz, $a = 1$ mm and $\epsilon_r = 15$, these are accurate up to resistivity about 2.4 $\Omega \cdot$cm, whereas at 3.2 GHz, the validity extends to resistivity about 1 $\Omega \cdot$cm for the same ϵ_r and the radius of the specimen.

The curves shown by full lines in Fig. 1 have been drawn from (17) and (18), for which the effective parameters have been calculated by assuming different values of ϵ_r and σ_r. However the evaluation of ϵ_r and σ_r from the measured values of $\epsilon_{r(eff)}$ and $\sigma_{r(eff)}$ requires the use of (21) and (23) which have been derived under the assumption that $A \gg 2B(\epsilon_r - \epsilon_{r(eff)})$. To check how accurately the values of ϵ_r and σ_r are obtained in actual cases, the values of $\epsilon_{r(eff)}$ and $\sigma_{r(eff)}$ reported by Champlin *et al.* and those obtained in the present measurements, have been used to calculate ϵ_r and σ_r from (21) and (23). The calculated values of ϵ_r and σ_r are tabulated in Table I. It may be seen that the values of ϵ_r calculated from (23) and (21) for Si and Ge samples used in the present studies show very good agreement with the literature values and the values of σ_r at 3.6986 GHz are close to the expected values calculated from their dc resistivity. Even in the case of the samples of wider range of resistivity used by Champlin *et al.*, the values calculated from the relations suggested in the present paper and those obtained from the complex transcendental equation, show an agreement within 1–2 percent for almost all the samples, although for the sample with resistivity 3 $\Omega\cdot$cm the agreement is slightly reduced. The reduced accuracy for very low resistivity samples ($\rho\sim1$ $\Omega\cdot$cm) is compensated by the ease with which σ_r and ϵ_r can be evaluated from (23) and (21).

ACKNOWLEDGMENT

The authors wish to thank Prof. G. P. Srivastava and Dr. M. K. Das for their suggestions and encouragement during the course of this work.

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Short Papers

Permittivity Measurement of Modified Infinite Samples by a Directional Coupler and a Sliding Load

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Abstract—**A cross coupler and waveguide sliding short technique for measuring the permittivity of an infinite sample is described in this paper. The experimental results obtained for commercially available cement, wheat flour, magnesium oxide, potassium bromide, glycerin, and water are given together with the estimated error. In view of the growing industrial use of microwaves, moisture dependent ϵ-values for cement and wheat flour are also reported.**

I. INTRODUCTION

A number of techniques for measuring the permittivity of dielectric materials at microwave frequencies have been reported [1] and a general review can be found in an excellent survey by Lynch [2]. These methods can be generally divided into three groups: transmission line methods, resonant methods, and perturbation methods [3]. The infinite sample method [4] is particularly suitable for routine measurements as a function of temperature. This is because of the convenience of temperature control and the simplicity with which the permittivity can be calculated from the experimental data. However, when the dielectric constant is large and/or the loss tangent is high, the uncertainty of measurements in conventional systems increases very rapidly. A method reported recently by Stuchly *et al.* [5] requires accurate measurement of the resonant frequency and the Q factor of the resonators for determining the permittivity of these materials.

The method presented in this paper calls for a four-port directional coupler and a precision sliding short. It has already been shown by the author [6] that the coupler and sliding short arrangement can be used for measurement in place of the slotted section. The dielectric constants of commercially available cement, wheat flour, magnesium oxide, potassium bromide, glycerin, and water are determined by this technique. Dependence of the permittivities of cement and wheat flour on moisture content is also studied by this method in X-band.

Manuscript received June 3, 1980; August 28, 1980.
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II. THEORETICAL RELATIONSHIPS

The input impedance of a homogenous nonmagnetic dielectric filled semi-infinite rectangular waveguide carrying TE mode is [5]

$$Z=\left[1-(\lambda/\lambda_c)^2\right]^{1/2}/\left[\epsilon-(\lambda/\lambda_c)^2\right]^{1/2} \quad (1)$$

where λ is the free space wavelength, λ_c is the cutoff wavelength

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